TRISEP 2013

COSMOLOGY & DARK MATTER

Part I

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Lecture I

- Our Universe: distances, age, homogeneity
- Hubble's law and scale factor
- Friedmann Lemaître equations
- Dynamics of the Universe
- Cosmological parameters & models

Lecture II

- Redshift and scale factor
- SN1a and the accelerating Universe
- Cosmic microwave background
- Baryo- / lepto genesis
- Inflation

Lecture III

- Large scale structure
- Neutralino interactions with matter
- Status of DM search experiments
- Future directions

THE UNIVERSE…..

The Universe is infinite and cyclic !
(Anaximander)

The Uiverse is finite, static and ever lasting.
(Aristoteles)

The Universe is a big rectangular box with Egypt at the center.
(the Egyptians)

The Universe is a gigantic vortex! (Aristophanes)

If God the Almighty would only have consulted me before creation, I would have proposed something simpler.
(A.de Rujula (CERN) ….around 1990)

The World was created on October 22, 4004BC at 6 o'clock in the evening.
(Bishop Usher 1650)

2013: Standard Model of Cosmology

69% dark energy, 26% dark matter, 5% ordinary matter; flat geometry

KEY EVENTS IN MODERN COSMOLGY

16th Century: N. Copernicus
Earth moves around $\odot$
…but still $\odot$ in the center of U.

18th Century: I. Newton
Stars are suns → static arrangement unstable

End of 18th Century: W. Herschel
Disk structure of Milky Way→ sun in the center!

Start of 20th Century: H. Shapley:
We are 2/3 away from gal. Center…but MW still at center of U.

Early 20th Century: Einstein, Friedmann, Lemaître, de Sitter → GR and dynamical models of the U.

1920 's: E. Hubble → Universe is expanding: v = H x D

1952: W. Baade: MW is an average galaxy → U looks the same for every observer(cosmological principle)

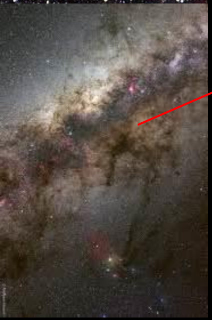
1960's: A Penzias, R. Wilson:
Discovery of 2.7K cosmic microwave background radiation

Today (WMAP, Planck): U. is 13.81 billion years old, with flat geometry and most of its mass is of un-known origin; the visible U. is only a fraction of the total U.

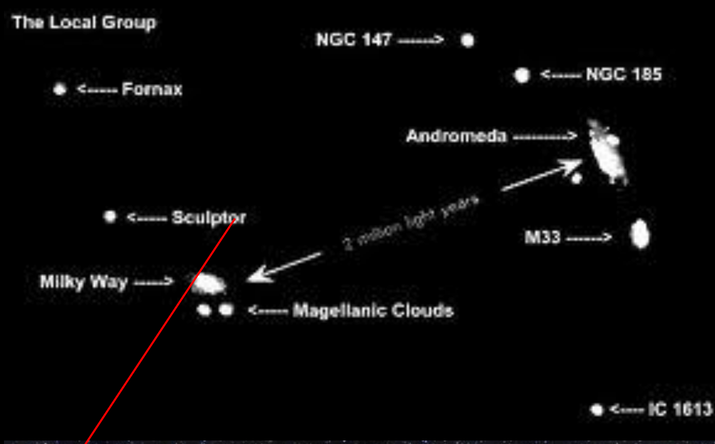
OUR UNIVERSE: DISTANCES

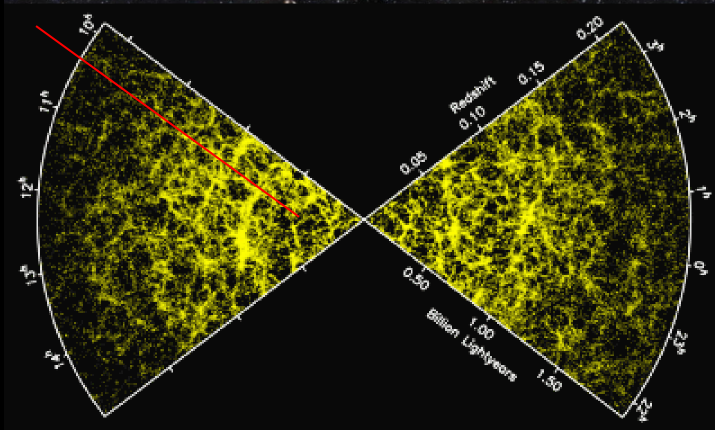
$1\ \text{pc} = 3.09 \times 10^{18}\ \text{cm} = 3.26\ \text{Ly}$

$(\text{Ly} = 9.46 \times 10^{15}\ \text{m})$

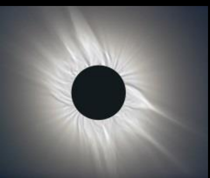


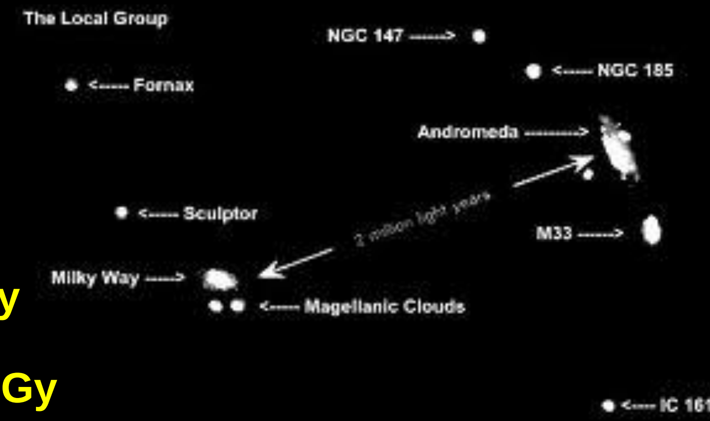
to the sun:	5 μpc
to nearest star (Prox. Cent.)	1 pc
to galactic centre	10 kpc
to gal. in local group (≈ 30)	50 – 100 kpc
to nearest cluster (Virgo ≈10^3)	50 Mpc
to scale of largest structures	100 Mpc
to « edge » of vis. Universe	14 Gpc





OUR UNIVERSE: How old is it?





Olber's paradox * : $t_U < 10^{23}$ **y**

Cosmochronology ($^{235}U/^{238}U$): $t_G \sim$ **6 Gy**

Stellar evol. in globular clusters: $t_{GC} >$ **11.2 Gy**

Ellip. galaxies large redshift: $t_{ell} = 13.4 \pm 1.4$ **Gy**

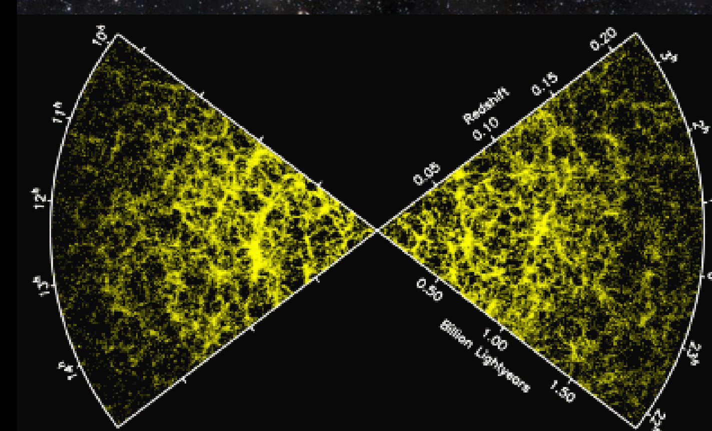
Age of U. 2013 (WMAP, Planck): $t_U = 13.81$ **Gy**

Olber's paradox * (1823):

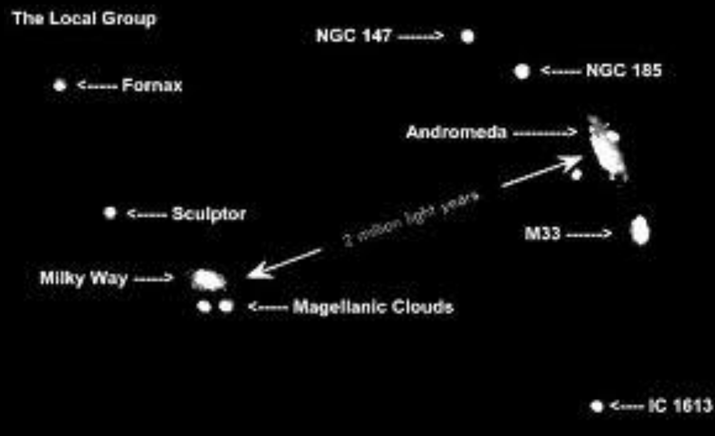
« Why is the night sky dark, if the U. is infinite, static and uniformly filled with stars? »

$$A/4\pi r^2 \cdot n \cdot 4\pi r^2\, dr \to \int A n dr \to \infty$$

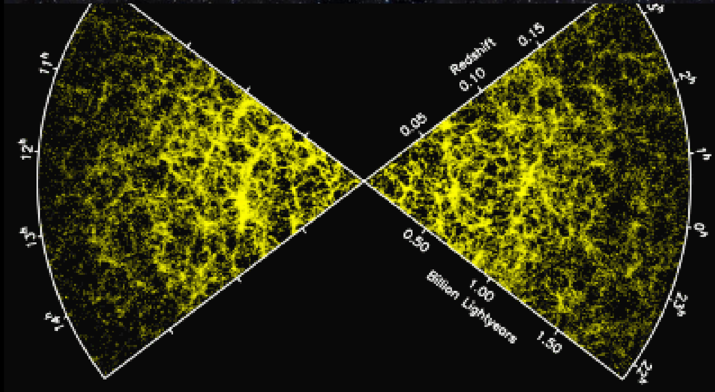
Answer: « Stars (gal.) had only finite time to radiate & exist only finite time & U. expanding;



OUR UNIVERSE: DENSITIES



Sun: ($M_\odot = 10^{33}$ g)	~ 1 - 100 g cm^{-3}
Neutron - star :	~ 10^{14} g cm^{-3}
Milky Way (10^{11} $M_\odot$):	~ 10^{-23} g cm^{-3}
Virgo cluster (10^{13} $M_\odot$) :	~ 10^{-29} g cm^{-3}
2.7 K CMB radiation	~ 10^{-34} g cm^{-3}
Avg. density of U.:	~ 10^{-30} g cm^{-3} * (~ 2 protons m^{-3})

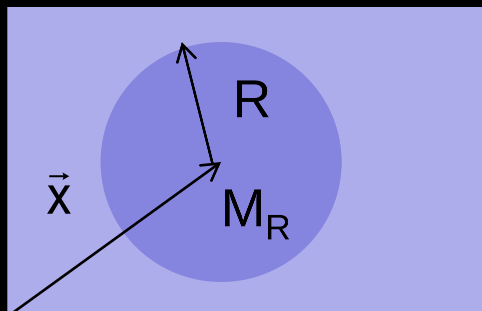


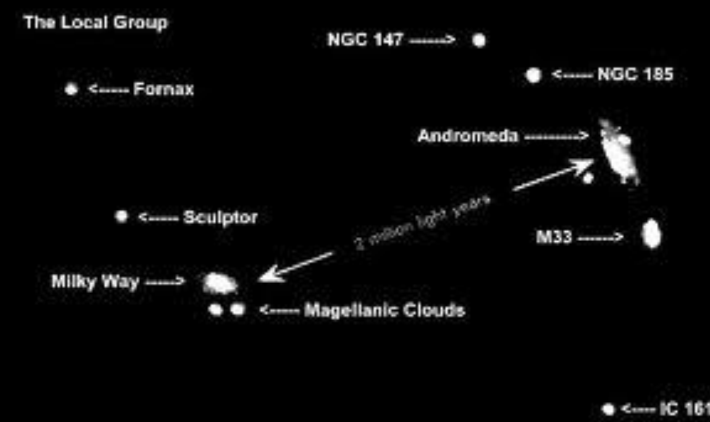
What is the « graininess » of the Universe?

*Best vacuum in lab: 10^{15} molecules m^{-3}

OUR UNIVERSE: HOMOGENEITY

Density fluctuations at different scales:





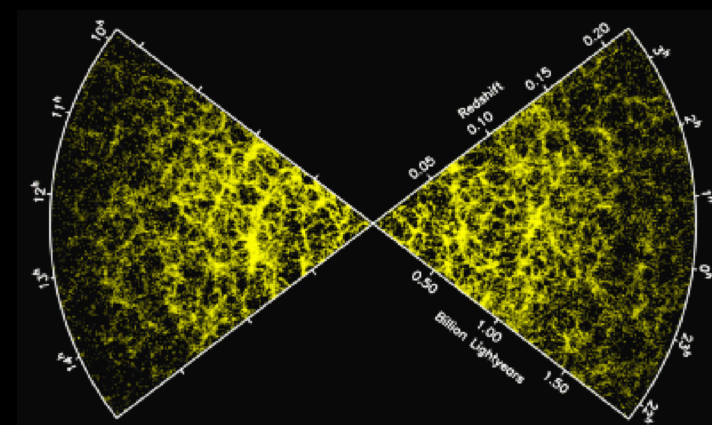
Define: $$\Delta = \left\langle \left(\frac{M(R,\vec{x})-\bar{M}}{\bar{M}}\right)^2 \right\rangle^{1/2}$$

R < 8 Mpc: $\Delta > 1$ **Small scale : « lumpy »**

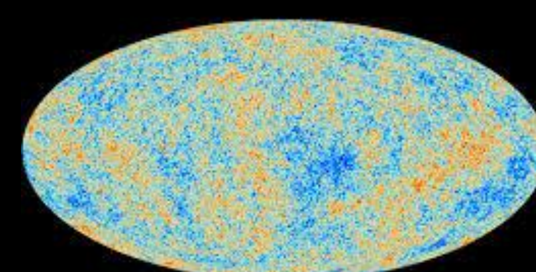
R ~ 8 Mpc: $\Delta \approx 1$

R ~ 0.6 Gpc: $\Delta \sim 10^{-1}$

R ~ 10 Gpc: $\Delta \sim 10^{-4}$ **Large scale: « homogenous »**



U. is homogenoeus and isotropic!



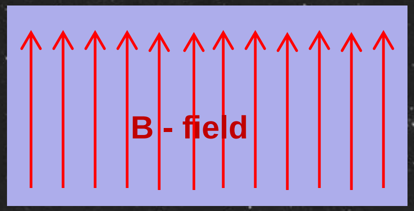
CMB: fluctuations at 10^{-5}

THE COSMOLOGICAL PRINCIPLE

« Viewed on a sufficiently large scale the properties of the U. are the same for all observers »

Homogeneity: U. looks everywhere the same

Isotropy: U. looks in every direction the same

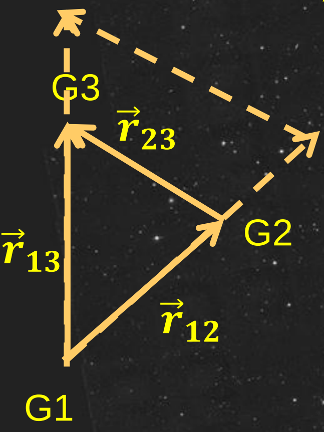


Homogeneous but not isotropical

Isotropical, but not homogeneous

Equal densities of galaxies in each direction:

$\rightarrow$ shape of triangle preserved in time



$$r_{ik} = |\vec{r}_i - \vec{r}_k| \qquad i,k = 1,2,3 \quad i \neq k$$

$$r_{ik}(t) = a(t) r_{ik}(t_0)$$

$a(t)$: « scale factor »

$$v_{ik}(t) = \dot{r}_{ik} = \frac{\dot{a}(t)}{a(t)} a(t) r_{ik}(t_0) = \frac{\dot{a}(t)}{a(t)} r_{ik}(t)$$

Define: $H(t) := \dfrac{\dot{a}(t)}{a(t)}$ $\Rightarrow$ $\boxed{v = H \cdot r \quad \textbf{....Hubble's Law !}}$

(small distances v<<c)

Edwin Hubble 1929

HUBBLE'S LAW

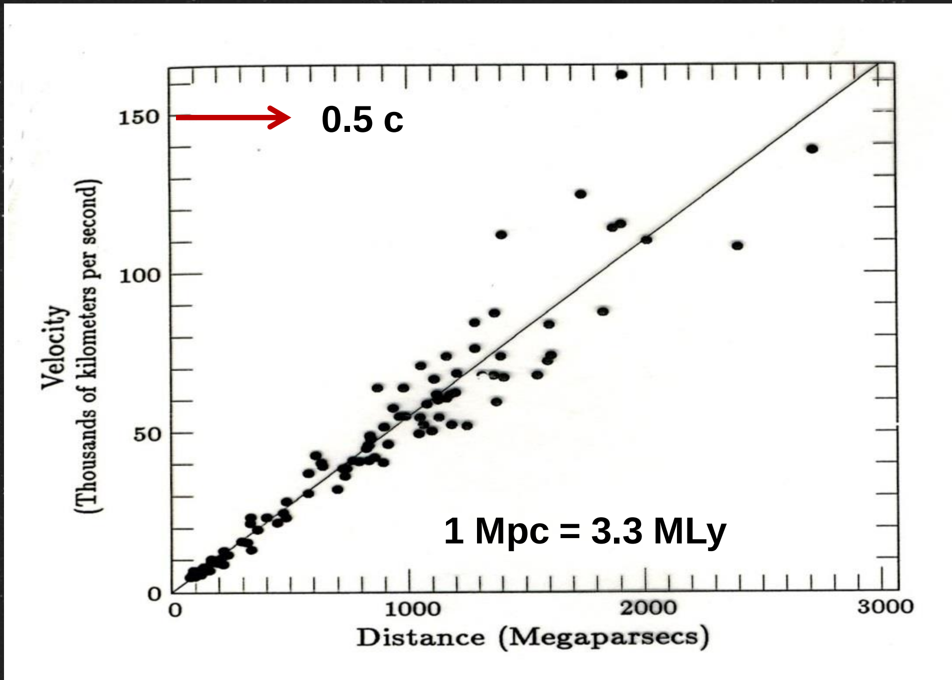
Redshift of 18 far away galaxies proportional to distance*

$$z = \frac{v}{c} = \frac{\lambda_0 - \lambda_e}{\lambda_e}$$

$(v << c)$

$$\boxed{v = H_0 D}$$

Hubble parameter in our epoch



$H_0 = 150 \pm 15$ (km/s)/Mpc (1929)*

$H_0 = 67.3 \pm 1.2$ (km/s)/Mpc (2013)

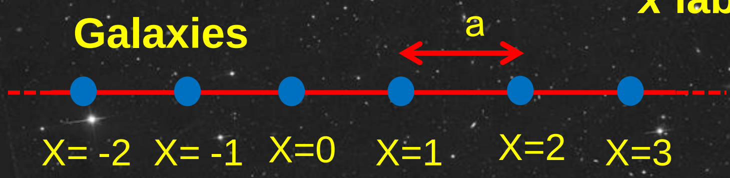
Expansion of the Universe → "Hubble Flow"

Hubble time : $t_H = H_0^{-1} = 14.5$ Gy ('13) → distance betw. "galaxies" → 0

*Mt. Wilson telescope

1920: Primordial Atom ? 1948: Steady State? 1949: Big Bang ?

HUBBLE'S LAW & SCALE FACTOR



***x* label position of galaxies → « comoving coordinates!**

$D(t) = a(t) \cdot \Delta x$ **distance betw. two galaxies** ***a*(*t*) → imagine an elastic thread**

Homogeneity o.k. ….but spaceing is time dependent!

Caveat: meter sticks do not expand!

$\dot{D}(t) = \dot{a}(t) \cdot \Delta x$ **Velocity of one galaxy rel. to another**

$$v = \dot{a}(t) \cdot \Delta x = \frac{\dot{a}}{a} \cdot a \cdot \Delta x = \frac{\dot{a}}{a} \cdot D = H \cdot D$$

Hubble's Law again!

$\boxed{\rightarrow H(t) \text{ …time dependent but independent of position!}}$

- everything squashed upon itself

a → 0 - density → ∞

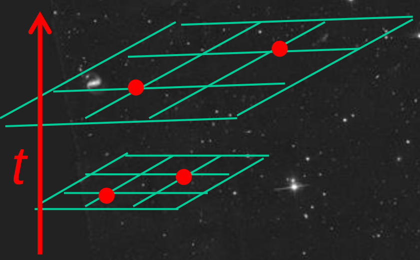
- Singularity happened everywhere

SCALE FACTOR & METRIC

Distance betw. two galaxies in flat 3D:

$$R(t) = a(t) \cdot \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

….or



$$ds^2 = a(t)^2(\Delta x^2 + \Delta y^2 + \Delta z^2)$$

Metric of flat 3D space

Same coord.

Add special relativity $\rightarrow$ **Minkowski space-time**

$$ds^2 = c^2 dt^2 - a(t)^2(\Delta x^2 + \Delta y^2 + \Delta z^2)$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -a^2 & 0 & 0 \\ 0 & 0 & -a^2 & 0 \\ 0 & 0 & 1 & -a^2 \end{pmatrix}$$

Metric of flat 3D space

…but curved space-time

$$ds^2 = c^2 d\tau^2$$

τ **proper time measured by a galaxy**

$$ds^2 = 0$$

For light rays

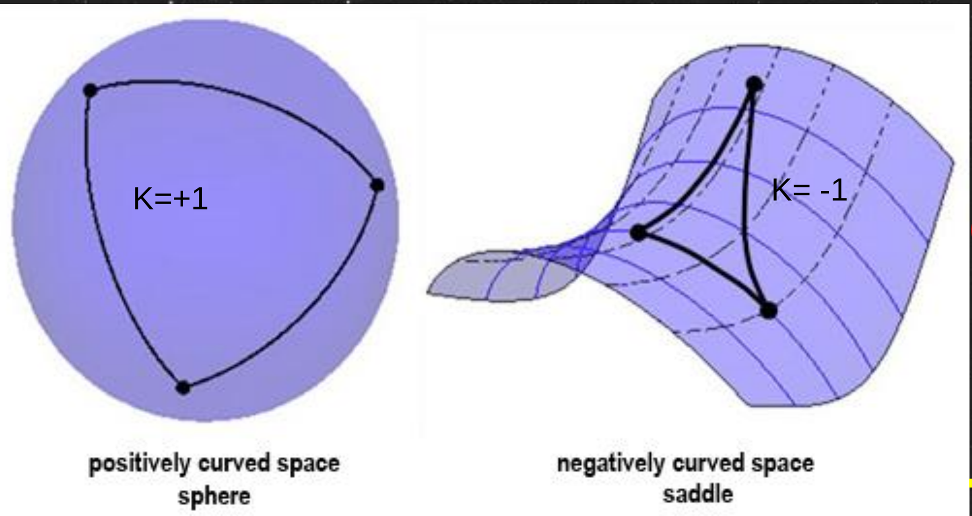
ROBERTSON-WALKER METRIC

RW metric $\rightarrow$ most general metric for flat or spatially curved homogeneous Universes

$$ds^2 = c^2dt^2 - a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2sin^2\theta d\varphi^2\right]$$

Recall: line element in flat 3D- space and spherical coordinates

$$ds^2 = dr^2 + r^2d\theta^2 + r^2sin^2\theta d\varphi^2$$



k = +1 positive spatial curvature

K = -1: negative curvature

k=0: flat space

~ our Universe
WMAP, Planck

…..Dynamics of the Universe?

GENERAL RELATIVITY & COSMOLOGY

Is GR important in cosmology?

Compare Schwarzschild radius $R_S = \frac{GM}{c^2}$ **with size of object**

	M	GM/c^2	L
Sun	$M_\odot$	1.3 km	3×10^6 km
Milky Way	$10^{12}\ M_\odot$	1.5×10^{12} km	3×10^{17} km
Universe	$\sim 10^{23}\ M_\odot$	$\sim 10^{23}$ km	$\sim 10^{23}$ km

Yes, GR is important!

NEWTONIAN DYNAMICS OF THE UNIVERSE

$$E_{kin} = \frac{m}{2}\dot{R}^2$$

$$E_{pot} = -G\frac{mM}{R}$$

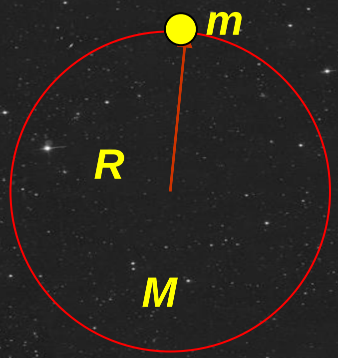
$$E_{kin} + E_{pot} = E_0$$

$$\frac{m}{2}\dot{R}^2 + \left(-\frac{GMm}{R}\right) = E_0$$

$$M = \rho\frac{4}{3}\pi R^3$$

$$\frac{m}{2}\dot{R}^2 - \frac{4\pi}{3}GR^2\rho m = E_0$$

Spherical region centered around any pt. in U….



M = mass inside *R*

m = mass of a border galaxy

(*M* : mass energy → mass + radiation + vac.energy)

U. is homogeneous → $\vec{R} = a(t)\vec{x}$ x : co-moving coord.; $\dot{x} = 0$

NEWTONIAN DYNAMICS OF THE UNIVERSE

$R = a(t)x$ x : co-moving coord.; $\dot{\boldsymbol{x}} = \boldsymbol{0}$ $\Rightarrow$ $\frac{m}{2}\dot{R}^2 - \frac{4\pi}{3}GR^2\rho m = E_0$

$$\frac{m}{2}\dot{a}^2x^2 - \frac{4\pi}{3}G\rho a^2x^2m = E_0 \ldots\ldots = -kmc^2x^2$$

in order to be true for all *x* (homogeneity)

$$\boxed{\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - kc^2}$$

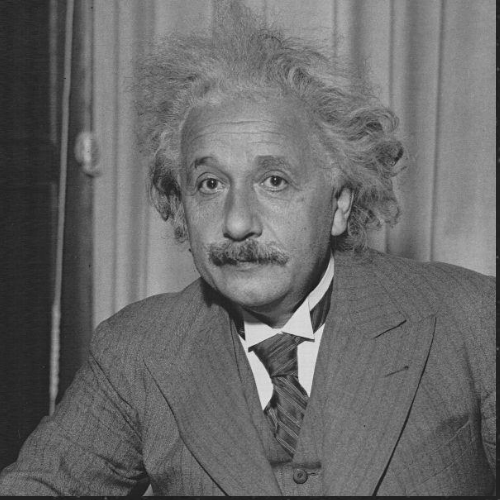
"Friedmann equation"

$kc^2 = \frac{-2E_0}{mx^2}$ Homogeneity: k = const.; E_0.= const. for a galaxy at x $\rightarrow E_0 \propto x^2$:

$k > 0$	$E_{kin} < E_{pot}$	expansion halts and reverses	closed U	
$k < 0$	$E_{kin} > E_{pot}$	expansion forever	open U	
$k = 0$	$E_{kin} = E_{pot}$	expansion halts for t $\rightarrow \infty$	flat U.	($v = v_{escape}$)

our Universe

EINSTEIN'S EQUATIONS



Albert Einstein 1915

Space-time tells mass-energy how to move*

Mass-energy tells space-time how to curve*

* J.A. Wheeler

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

16 equations

Ricci tensor with 1st & 2nd derivatives of $g_{\mu\nu}$

Ricci scalar $R = g^{\mu\nu} R_{\mu\nu}$

Energy-momentum tensor

$$T_{\mu\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

Perfect fluid of matter and radiation in spatially flat space

Energy density

Pressure (adds to gravity!)

Caveat: for cosmologists energy density = mass density $\rightarrow \varepsilon = \rho c^2 \rightarrow \varepsilon = \rho \rightarrow c = 1$

THE FRIEDMANN - LEMAÎTRE EQUATIONS

…are the solutions of Einstein's equations: **(c = 1!)**

F1* $$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - k$$ **Isaac = Einstein!**

→ Rate of cosmic expansion increases with ρ

F2 $$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3p)a$$ **Acceleration equation**

→ Acceleration decreases with increasing ρ and p

- A.E. did not believe these equations because they did not allow for a static Universe
- confirmed by cleric G. Lemaître (1922)
- only accepted by A.E. after Friedmanns death

*Alexander Friedman 1922

THE COSMOLOGICAL CONSTANT

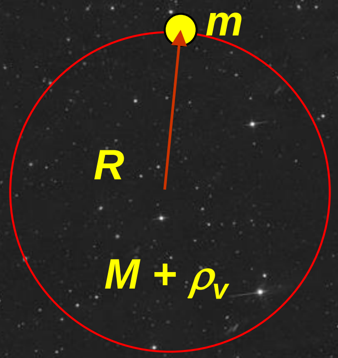
…introduced by A.E. to stop contraction*

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu}$$

Plays the role of a «constant vacuum energy density »
(…today called dark energy)

$$E_v = \rho_v c^2 = \frac{c^4}{8\pi G} \Lambda$$

Mass - energy gravitates!



$$E_{pot} = -G \frac{mM}{R}$$

$$E_{pot}{}^{v} = -G \frac{m}{R} \left(\rho_v \frac{4}{3} \pi R^3 \right)$$

$$F = -\frac{dE_{pot}}{dR} = -G \frac{Mm}{R^2} + \frac{1}{3} m \Lambda c^2 R$$

attractive

$\Lambda > 0$: repulsive

* A.E. later: ''the biggest blunder of my life''

Λ adds to gravitation (±)

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - k$$

THE COSMOLOGICAL FLUID

To solve Friedmann's equations $\rightarrow$ know evolution of ρ(t) and ρ(p)!

Treat U. as a cosmic fluid of matter, radiation and dark energy

1st law of thermodynamics:

$$dE = T\cancel{ds} - p\,dV \qquad E = \frac{4}{3}\pi a^3 \rho$$

Volume of co-moving radius r =1

$$\frac{dE}{dt} = -p\frac{dV}{dt} \qquad \frac{d}{dt}(\rho a^3) = -p\frac{d}{dt}a^3$$

$$\boxed{\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0}$$

« Fluid equation »

$$\boxed{p = \omega \cdot \rho}$$

(Equation of state)

Matter:	$p = 0$	$\omega = 0$	$\rho \propto a^{-3}$
Radiation:	$p = \rho/3$	$\omega = 1/3$	$\rho \propto a^{-4}$
Vac. energy:	$p = -\rho$	$\omega = -1$	$\rho =$ const.

…what is negative pressure?

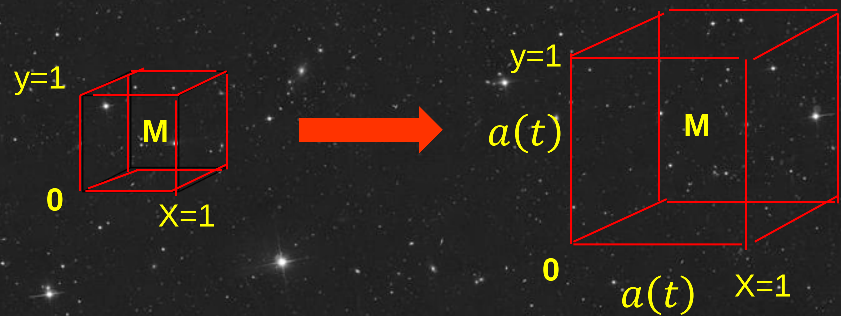
DYNAMICS OF THE UNIVERSE: MATTER

A quite realistic example:

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_m a^2$$

Define $a_0 = 1$ for our epoch t_0

k = 0; matter dominated



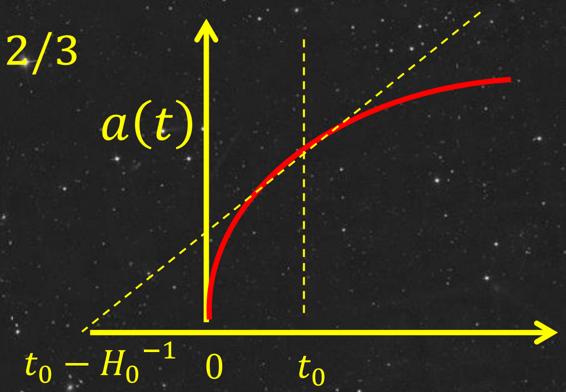
Our epoch $\rho_{0,m}$

later… $\rho_m = \frac{\rho_{0,m}}{a^3}$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_{0,m} a^{-1}$$

> 0 → U. either grows or shrinks …. always

…try: $a(t) = \alpha t^\beta$ → $a(t) = \left(\frac{t}{t_0}\right)^{2/3}$



$$\frac{\dot{a}}{a} = H(t) = \frac{2}{3t}$$

→ **t_U = 9.7 Gy**

H_0= 67.3 km/s/Mpc

Not bad for first estimate ! (→13.8 Gy)

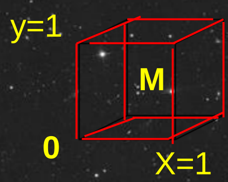
DYNAMICS OF THE UNIVERSE : RADIATION

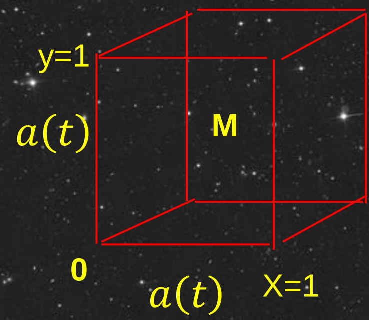
A quite realistic example:

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_r a^2$$

Define a_0 = 1 for our epoch

k = 0; radiation dominated





Our epoch $\rho_{0,r}$

later…

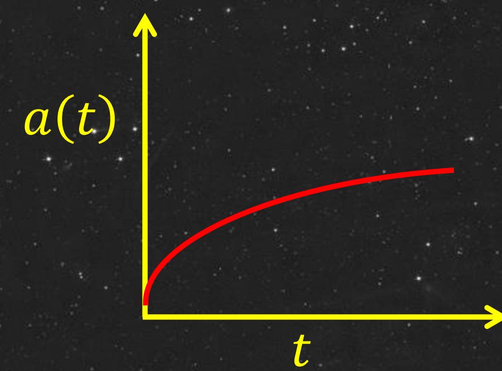
$$\rho_r = \frac{\rho_{0,r}}{a^3 \cdot a} = \frac{\rho_{0,r}}{a^4}$$

Wavelength $\lambda \propto a$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_{0,m} a^{-2}$$

> 0 $\rightarrow$ U. either grows or shrinks …. always

…try: $a(t) = \alpha t^{\beta}$ $\Rightarrow$ $a(t) = \left(\frac{t}{t_0}\right)^{1/2}$



$$\frac{\dot{a}}{a} = H(t) = \frac{1}{2t}$$ $\Rightarrow$ **t_U = 7 Gy**

H_0= 67.3 km/s/Mpc

Quite a bit off! ($\rightarrow$13.8 Gy)

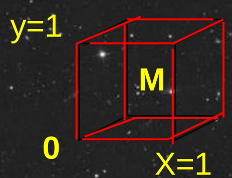
DYNAMICS OF THE UNIVERSE : VACUUM ENERGY

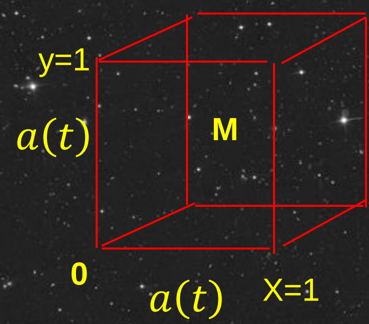
A quite realistic example:

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_v a^2$$

Define a_0 = 1 for our epoch

***k* = 0; dark energy dominated**





Our epoch: $\rho_{0,v}$

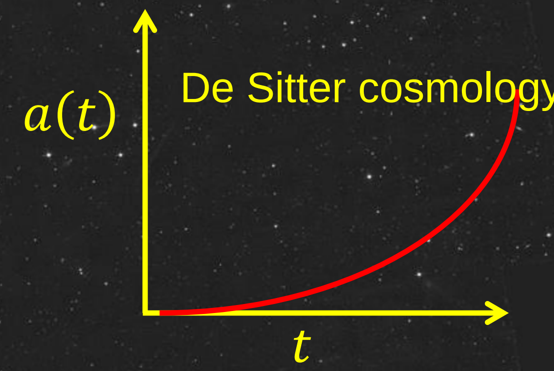
later… $\rho_v = \rho_{0,v} = const.$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_{0,v} a^2$$

$$\rho_v = \frac{\Lambda c^2}{8\pi G}$$ Einstein's cosmol. constant

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{1}{3}\Lambda c^2 \Rightarrow \dot{a}(t) = \sqrt{\Lambda/3}\, a(t)$$

$$a(t) = a_0 exp\left(\sqrt{\Lambda/3}\, t\right) = a_0 e^{Ht}$$



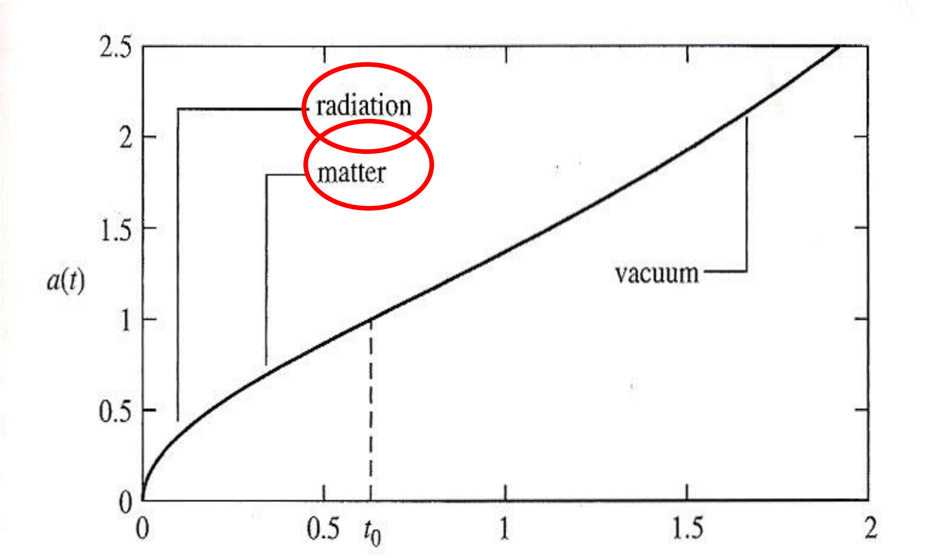
The future of our U.!

DYNAMICS OF THE UNIVERSE: DENSITIES

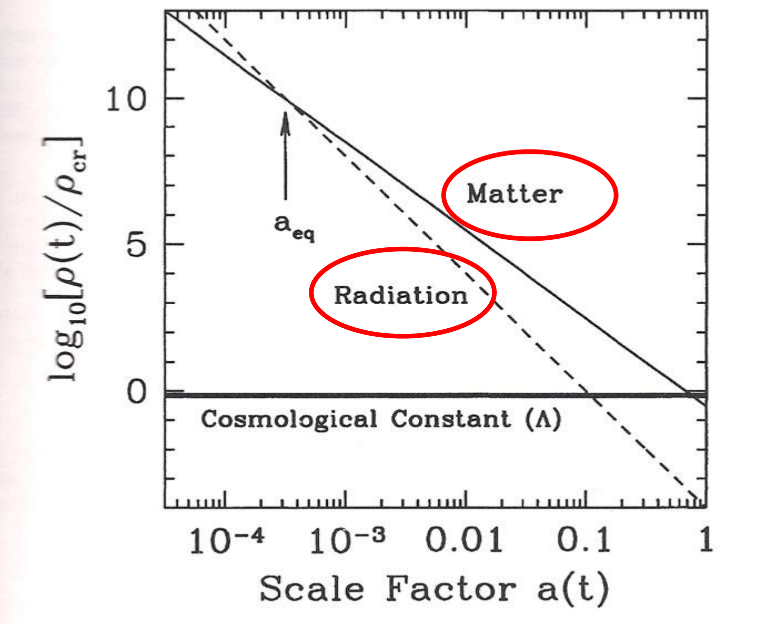
Today : $\rho_{tot}(t_0) \sim 10^{-30} g cm^{-3}$ $\quad \frac{\rho_v}{\rho_{tot}} = 0.7$ $\quad \frac{\rho_m}{\rho_{tot}} = 0.3$ $\quad \frac{\rho_r}{\rho_{tot}} \sim 10^{-5}$

$\rho_m(t) \propto a^{-3}$ $\qquad \rho_r(t) \propto a^{-4}$ $\qquad \rho_v(t) = const.$

Radiation:	$a(t) \propto t^{1/2}$	$\rho_r(t) \propto t^{-2}$	$\rho_m(t) \propto t^{-3/2}$	$\rho_v(t) = const.$
Matter:	$a(t) \propto t^{2/3}$	$\rho_m(t) \propto t^{-2}$	$\rho_r(t) \propto t^{-8/3}$	$\rho_v(t) = const.$
Dark energy:	$a(t) \propto e^{Ht}$	$\rho_m(t) \propto e^{-3Ht}$	$\rho_r(t) \propto e^{-4Ht}$	$\rho_v(t) = const.$



$a(t_0) = 1$



CRITICAL DENSITY

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_{tot}a^2 - k \quad \rightarrow \quad \left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho_{tot} - \frac{k}{a^2}$$

$$\rho_{tot} = \rho_m + \rho_r + \rho_v$$

For a flat U. k = 0

$$\boxed{\rho_c(t) = \frac{3H(t)^2}{8\pi G}}$$

today: 1.9×10^{-29} gm^{-3} (~2 p/m^3)

from H_0 = 67.3 kms/Mpc

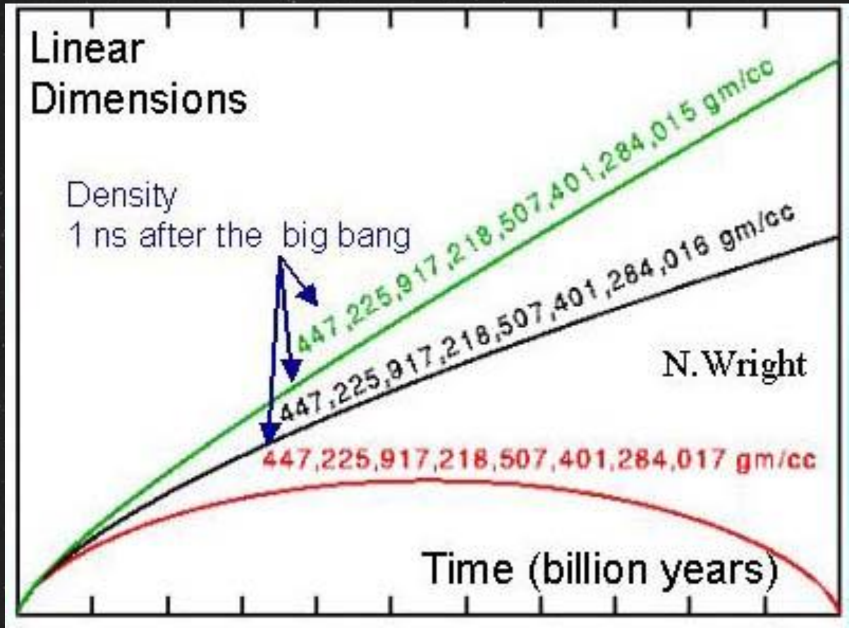
$\rho_{tot} = \rho_c$ flat U. (Einstein-deSitter)

$\rho_{tot} > \rho_c$ closed U.

$\rho_{tot} < \rho_c$ open U.

Note: - U with $\rho_c \rightarrow$ forever with ρ_c !

- ρ_c changes with time



THE COSMOLOGICAL PARAMETERS

Define: $\Omega_i(t) = \frac{\rho_i(t)}{\rho_c} \quad i = m, r, v$

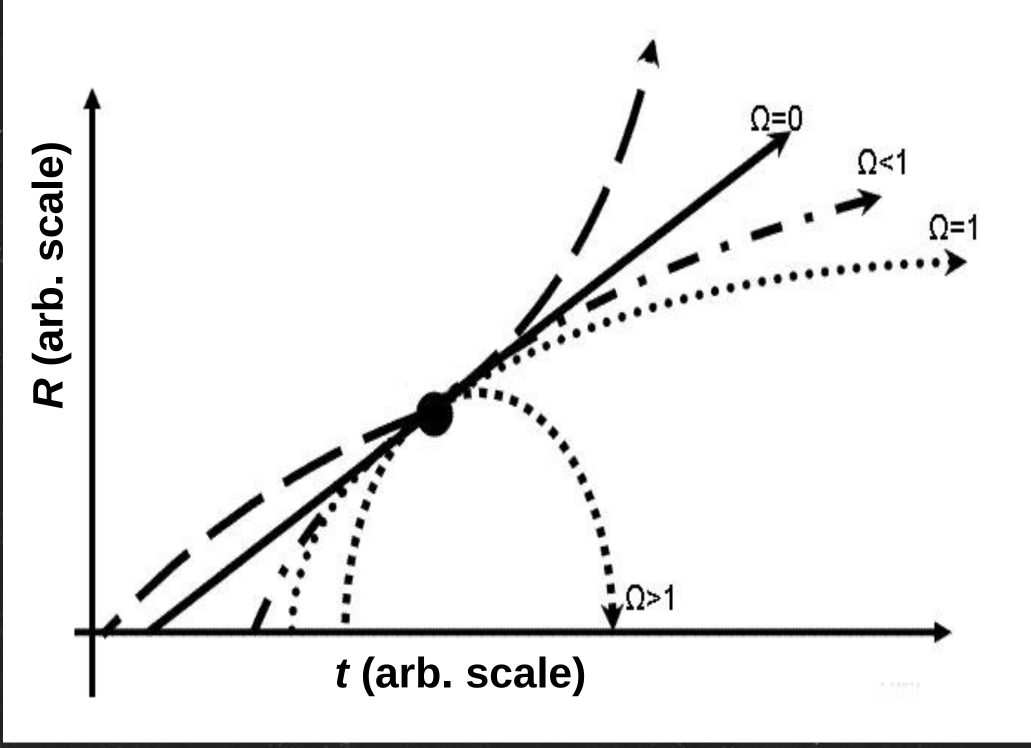
$$\boxed{\Omega_{tot} = \Omega_m + \Omega_r + \Omega_v}$$

Together with $H(t)$ called "cosmological parameters"

$\Omega_{tot} > 1$ closed U.

$\Omega_{tot} < 1$ open U.

$\Omega_{tot} = 1$ flat, "critical U."



Fate of U. depends on the cosmological parameters!

…how to quantify this?

THE COSMOLOGICAL EQUATION

- Relates today's observed parameters with those in the past and future
- Describes all possible cosmological models
- One of the most important equations in cosmology

Step 1: Friedmann again

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_{tot}a^2 - k \qquad \rho_c(t) = \frac{3H(t)^2}{8\pi G}$$

$$\dot{a}^2 = H^2a^2 = H^2a^2\Omega_{tot} - k$$

Step 2: Find k from todays parameters H_0, $\Omega_{m,0}$, $\Omega_{\Lambda,0}$

$$k = H^2(t)a^2(t)[\Omega_{\boldsymbol{tot}}(t) - 1] = {H_0}^2{a_0}^2[\Omega_{tot,0} - 1]$$

k const. all times

measured

def.: =1

measured

THE COSMOLOGICAL EQUATION

Step 3: Find evolution of $\Omega_i(t)$ from today's parameters $\Omega_{i,0}$

e.g.: $\Omega_{m,0} = \frac{\rho_{m,0}}{\rho_{c,0}} = \frac{8\pi G}{3{H_0}^2}\rho_{m,0}$ ➡ $\Omega_m(t) = \frac{\rho_m}{\rho_c} = \frac{8\pi G}{3H^2}\rho_{m,0}a^{-3} = \left(\frac{H_0}{H}\right)^2 \Omega_{m,0}a^{-3}$

Equation of state!

$$\Omega_i(t) = \left(\frac{H_0}{H(t)}\right)^2 \Omega_{i,0}\, a(t)^{-3(1+\omega_i)}$$

$\omega_m = 0$

$\omega_r = 1/3$

$\omega_v = -1$

Step 4: Insert into Friedmann equation

$$\dot{a}^2 = {H_0}^2\left(\Omega_{m,0}a^{-1} + \Omega_{r,0}a^{-2} + \Omega_{\Lambda,0}a^2 + 1 - \Omega_{tot,0}\right)$$

- Now we can predict the evolution of the U. from the parameters observed today!
- Wide variety of models depending on Ω_m , Ω_Λ , k

$\rightarrow$ *+ still one other usefull parameter!*

THE DECELERATION PARAMETER

…measures the change of the expansion rate* $\rightarrow$ expansion of $a(t)$ around t_0

$$a(t) = a(t_0)\left(1 + H_0\Delta t \ + q_0 {H_0}^2 \Delta t^2 \ldots\right)$$

$$\boxed{q_0 = -\frac{\ddot{a}(t_0)a(t_0)}{\dot{a}^2(t_0)}}$$

dimensionless

Acceleration equation: $\ddot{a} = -\frac{4\pi G}{3}(\rho + 3p)a = -\frac{4\pi G}{3}(1 + 3\omega)\rho a$

equation of state

$$\boxed{q_0 = \frac{1}{2}\sum_i \Omega_{i,0}(1 + 3\omega_i)}$$

$k = 0$

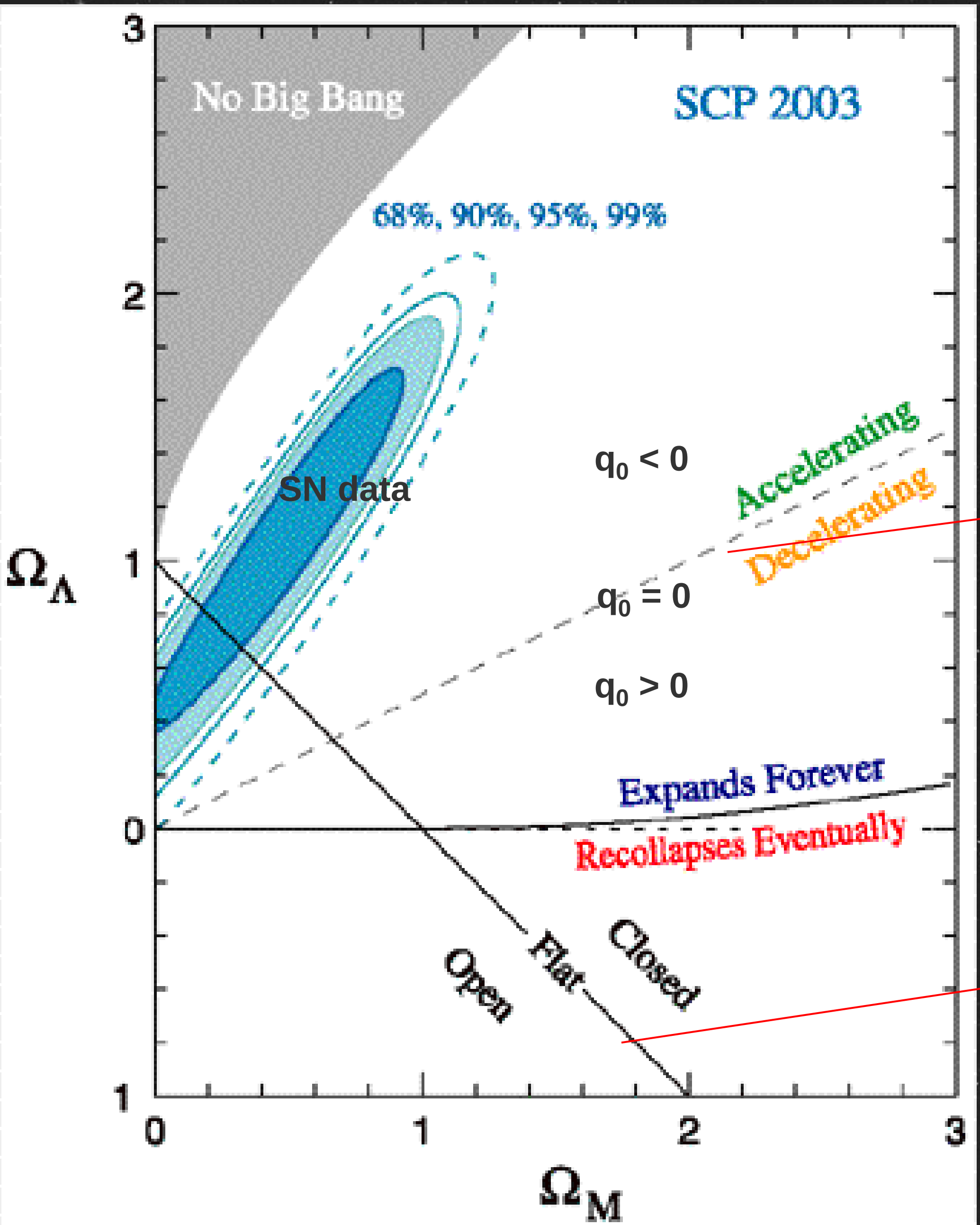
$q_0 < 0$ acceleration

$q_0 > 0$ deceleration

Acceleration for $\omega_i > -1/3$

*Remember: the Hubble parameter must not be constant!!!

COSMOLOGICAL MODELS



We know: $\Omega_{r,0} \approx 0$ today

$$q_0 = \frac{1}{2}\{\Omega_{m,0} - 2\Omega_{\Lambda,0}\}$$

Dividing line "accel." / "decel."

$$\boxed{\Omega_{\Lambda,0} = \frac{1}{2}\Omega_{m,0}}$$

$$\Omega_{tot} = \Omega_m + \Omega_v = 1 \quad (k = 0)$$

Dividing line "open" / "closed"

$$\boxed{\Omega_{\Lambda,0} = 1 - \Omega_{m,0}}$$

How to measure Ω_i ?